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A statistical approach to determination of a mineral lineation

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Abstract

A statistical model of the bipolar von Mises type was applied to the analysis of mineral lineation defined by the preferred dimensional orientation of inequant grains on a foliation surface. Orientation data of 203–258 grains from each of four rock specimens fit the model well, suggesting a useful analytical technique with three advantages: (1) confidence in whether a metamorphic tectonite has a mineral lineation can be determined using a χ^2-test, (2) the orientation of the mineral lineation can be represented by the mean direction ($\bar{\theta}$) with a specified confidence interval, and (3) the degree of preferred dimensional orientation of grains can be quantified by the concentration parameter ($\hat{\kappa}$).

1. Introduction

High grade metamorphic tectonites commonly possess a mineral lineation defined by preferred dimensional orientation of inequant grains on the foliation surface (e.g. Hobbs et al., 1976; Passchier and Trouw, 1996). If most grains are arranged in a similar direction, we can reliably determine orientation of the lineation in the field. However, not all tectonites possess a 'strong' mineral lineation. Schists or gneisses with variably oriented inequant grains may be encountered for which the direct measurement of the lineation may be difficult and unreliable. In such cases, structural geologists try to ascertain whether a mineral lineation is present or not, and then try to measure its orientation. Our judgement largely depends on our intuition. This is neither satisfactory nor quantifiable. An objective basis is needed to determine whether such a structure is present and then to measure its orientation.

We propose a statistical method that resolves this problem, and as examples we analyse four specimens by this method. Since we deal with two-dimensional orientation data, we use a von Mises distribution (e.g. Mardia, 1972; Batschelet, 1981).

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2. The von Mises distribution

We assume that our orientation data (θ_i: $i = 1 \ldots n$) were drawn at random from a bipolar von Mises type distribution (e.g. Mardia, 1972; Batschelet, 1981), because we restrict θ to the range $0^\circ \le \theta_i \le 180^\circ$. The number of data is shown by n. The assumption is tested later. The probability density function is given by

$$f(\theta) = \frac{1}{\pi I_0(\kappa)} \exp[\kappa \cos 2(\theta - \mu_0)] \tag{1}$$

where θ is a circular variable and $I_0(\kappa)$ is a modified Bessel function of the first kind, i.e.

$$I_0(\kappa) = \sum_{r=0}^{\infty} \frac{1}{r!^2} \left(\frac{\kappa}{2}\right)^{2r}. \tag{2}$$

The parameters are μ_0 the mean direction and κ the concentration. The probability density function is satisfied by the equation

$$1 = \int_{0^\circ}^{180^\circ} f(\theta)\, \mathrm{d}\theta. \tag{3}$$

We estimate the mean direction ($\bar{\theta}$) and the mean resultant length ($\bar{R}$) from our data (θ_i) as

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$$\bar{\theta} = \frac{1}{2}\arctan\frac{\sum_{i=1}^{n}\sin 2\theta_i}{\sum_{i=1}^{n}\cos 2\theta_i} \tag{4}$$

and

$$\bar{R} = \frac{1}{n}\sqrt{\left(\sum_{i=1}^{n}\sin 2\theta_i\right)^2 + \left(\sum_{i=1}^{n}\cos 2\theta_i\right)^2}, \tag{5}$$

respectively. $\bar{\theta}$ is the same as the azimuth of the resultant vector or the vector mean, whereas $\bar{R}$ is the same as the magnitude of the resultant vector or the vector magnitude (Curray, 1956).

The maximum likelihood estimates (e.g. Mardia, 1972, p. 122) of the mean direction ($\hat{\mu}_0$) and of the concentration parameter ($\hat{\kappa}$) are given as

$$\hat{\mu}_0 = \bar{\theta} \tag{6}$$

and as the solution of

$$\frac{I_1(\hat{\kappa})}{I_0(\hat{\kappa})} = \bar{R}, \tag{7}$$

respectively, where $I_1(\hat{\kappa})$ is a modified Bessel function of the first kind, i.e.

$$I_1(\hat{\kappa}) = \sum_{r=0}^{\infty}\frac{1}{r!^2(r+1)}\left(\frac{\hat{\kappa}}{2}\right)^{1+2r}. \tag{8}$$

$\hat{\kappa}$ is a monotonic function of $\bar{R}$; $\bar{R}$ ranges from 0 to 1, while $\hat{\kappa}$ ranges from zero to ∞. If $\bar{R}=0$, then $\hat{\kappa}=0$. This case shows the uniform distribution of θ_i. If $\bar{R}=1$, then $\hat{\kappa}=\infty$. This case shows that all θ_i are the same. The relationship between $\bar{R}$ and $\hat{\kappa}$ is tabulated in Mardia (1972, p. 298). $\hat{\kappa}$ is also approximated to two significant figures by the following expressions (Mardia, 1972, pp. 122–123):

$$\hat{\kappa} = \frac{\bar{R}}{6}(12 + 6\bar{R}^2 + 5\bar{R}^4)\ \ (\bar{R}<0.65) \tag{9}$$

$$\frac{1}{\hat{\kappa}} = 2(1-\bar{R}) - (1-\bar{R})^2 - (1-\bar{R})^3\ \ (\bar{R} > 0.65). \tag{10}$$

μ_0 is estimated as

$$\bar{\theta} - d_0 < \mu_0 < \bar{\theta} + d_0 \tag{11}$$

where d_0 is a confidence interval of the mean direction (e.g. Mardia, 1972, p. 302; Cheeney, 1983, p. 101). The confidence interval is tabulated at the various values of the critical region (Mardia, 1972, p. 298), and also approximated for practical purposes by

$$d_0 = \frac{112}{\sqrt{n\bar{R}\hat{\kappa}}} \tag{12}$$

at the critical region of 0.05 (Cheeney, 1983, p. 101). The range of the concentration parameter (κ) is given as

$$\kappa_1 < \kappa < \kappa_u \tag{13}$$

where κ_1 and κ_u are the lower and upper confidence

Table 1
Summary of the specimens and the results

Sample and locality	Lithology (Metamorphic grade)	Columnar mineral	Matrix minerals	Tectonic unit	$\bar{\theta}$ $\bar{R}$	d_0 $\hat{\kappa}$	Selected references
A: Wadi Tayin, Oman	metachert (amphibolite)	piedmontite	quartz, muscovite, albite, epidote, apatite, opaque	Metamorphic sole of the Oman ophiolite	40° 0.18	28° 0.36	(1), (2), (3)
B: Besshi, Japan	metabasite (epidote–amphibolite)	hornblende	epidote, chlorite, albite, muscovite, quartz, apatite, opaque	Sambagawa metamorphic belt	135° 0.10	55° 0.20	(4), (5), (6)
C: As Sifah, Oman	metabasite (eclogite)	glaucophane	omphacite, garnet, epidote, albite, chlorite, quartz, opaque	Saih Hatat window	120° 0.38	14° 0.82	(7), (8), (9)
D: Wakayama, Japan	metachert (glaucophane schist)	glaucophane	quartz, muscovite, epidote, albite, apatite, opaque	Sambagawa metamorphic belt	131° 0.85	4° 3.7	(10), (11), (12)

(1) Boudier and Coleman (1981), (2) Ghent and Stout (1981), (3) Hacker and Mosenfelder (1996), (4) Higashino (1990), (5) Otsuki and Banno (1990), (6) Takasu and Makino (1980), (7) El-Shazly and Coleman (1990), (8) El-Shazly et al. (1997), (9) Searle et al. (1994), (10) Hirota (1991), (11) Kanehira (1967), (12) Wang and Maekawa (1997).

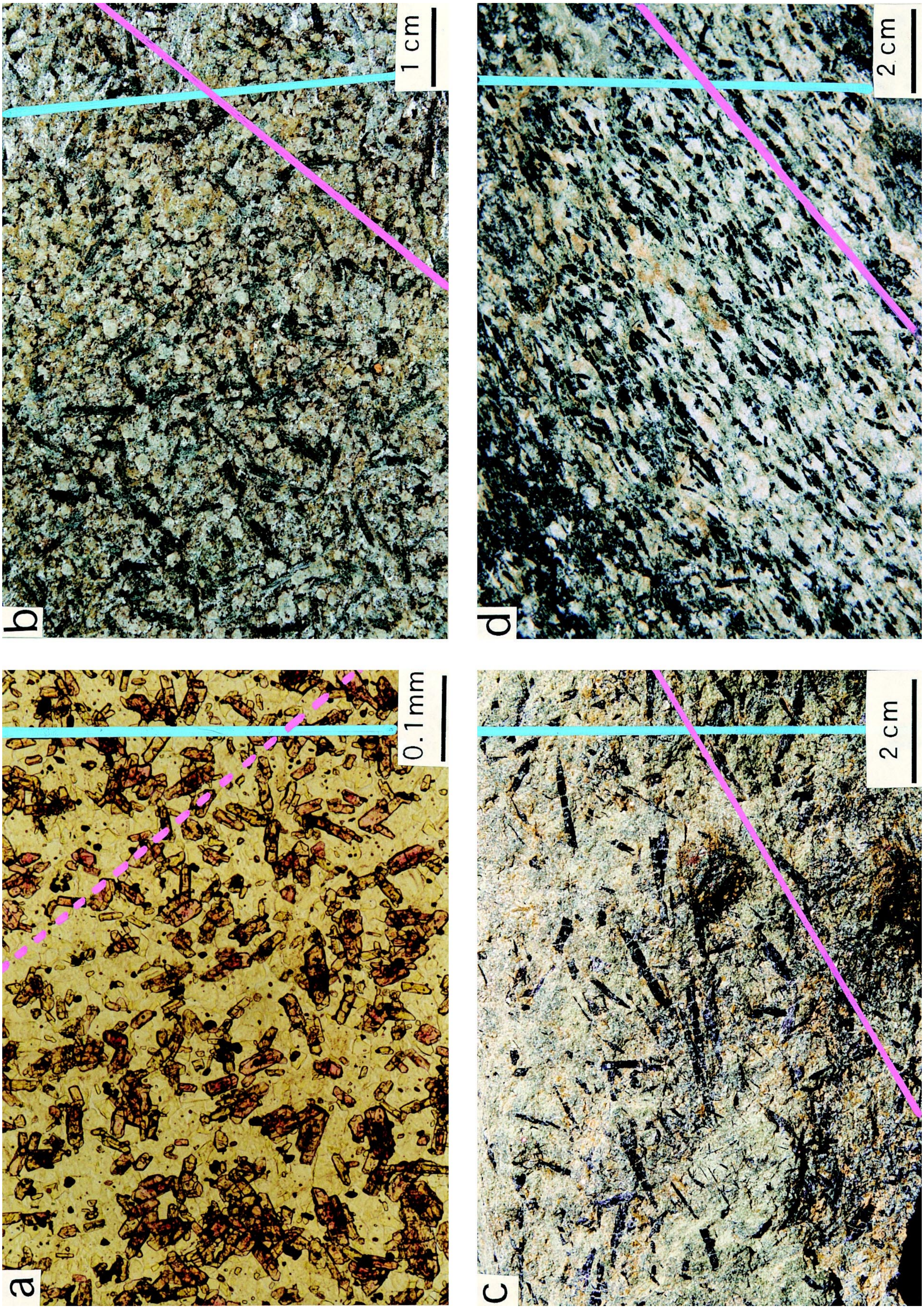

Fig. 1. Photographs (a–c) and photomicrograph (d) of columnar grains on foliation surfaces. Specimens A–D correspond to (a)–(d), respectively. The reference line is drawn in blue and the mean direction in pink. The mean direction in (a) is in a broken line, because specimen A could not be fitted by a von Mises distribution. The orientation was measured anticlockwise from the reference line as the angle to the longest axis of a grain.

limits, respectively, delineated at the critical region of 0.10 by Mardia (1972, p. 304). κ_l and κ_u are also estimated by the bootstrap method (Fisher, 1993).

3. Specimens

To illustrate the technique, we chose four unrelated schist specimens from Japan and Oman (A–D). A brief description is given in Table 1. They have a foliation defined by planar arrangement of platy minerals and compositional layering. Columnar grains occur on the foliation surface, their longest axes being variably oriented. The degree of the preferred dimensional orientation appears different from specimen to specimen (Fig. 1).

Specimen A appears to have no observable lineation with randomly oriented piedmontite grains in a quartz matrix (Fig. 1a). Glaucophane grains in specimen B (Fig. 1b) show a range of orientations making it highly difficult for us to confidently determine the preferred orientation by eye. Specimen C (Fig. 1c) shows variably oriented glaucophane grains. However, the existence of a lineation and its measurement can be achieved by routine visual examination. Specimen D exhibits a strong parallel orientation of glaucophane grains (Fig. 1d) that unequivocally depict a lineation.

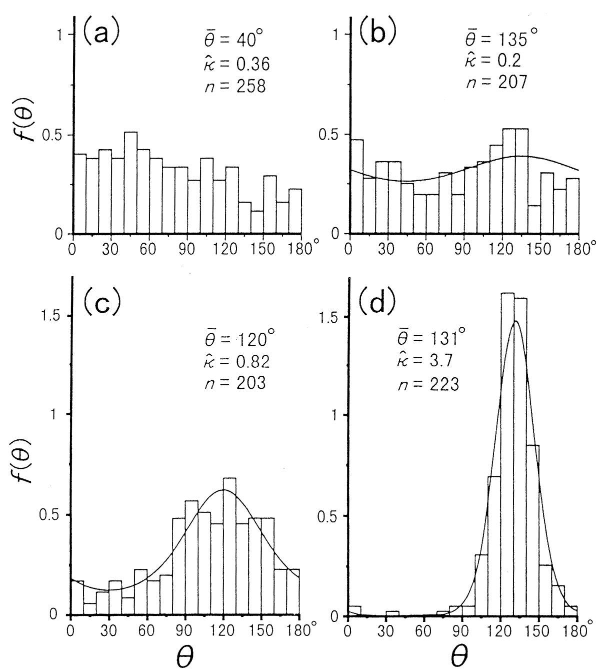

Fig. 2. Distributions of orientation data. Parts (a)–(d) show the histograms of the orientation data of specimens A–D, respectively. Solid line in (b)–(d) shows the probability density function of the von Mises distribution. See text for discussion.

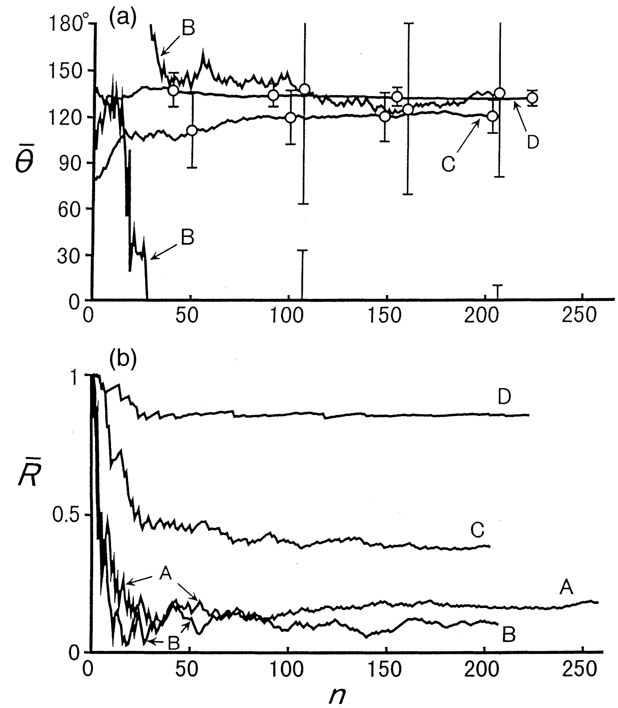

Fig. 3. CUSUM plots (Fisher, 1993) showing increasing stability of $\bar{\theta}$ (a) and $\bar{R}$ (b) with increasing n. Error bars in (a) show 95% confidence intervals. See text for discussion.

4. Measurement and analysis

1. We measured the orientation (θ_i) of the longest axes of the 203–258 columnar grains on photographs (photomicrographs) of the foliation surface. As the reference line from which the orientation was measured is not important in our case, we arbitrarily drew a line on the photographs. Some glaucophane grains seem to be arranged obliquely to the foliation plane. Such grains were projected onto the foliation plane and the orientation of the apparent longest axis was measured.
2. A histogram of θ_i was made, each class having width 10° (Fig. 2).
3. A χ^2-test (e.g. Cheeney, 1983, pp. 50–52) was adapted to test for uniformity with critical region of 0.05. χ^2 is calculated as

$$\chi^2 = \sum_{j=1}^{18} \frac{(f_j - f_j^*)^2}{f_j^*} \tag{14}$$

where f_j ($j = 1 \ldots 18$) is the frequency of each class of the histogram of θ_i and f_j^* ($j = 1 \ldots 18$) is the expected frequencies of reference distribution. For the uniform distribution, $f_j^* = n/18$ ($j = 1 \ldots 18$). The critical values of χ^2 are tabulated in, e.g. Cheeney (1983, p. 56). There are 17 degrees of freedom. The null hypothesis that the distribution is uniform was rejected for specimens B–D but not rejected for specimen A. Hence, we conclude that specimen A contains no mineral lineation on the foliation plane.

4. We obtained $\bar{\theta}$ and $\bar{R}$ by substituting θ_i into Eqs. (4) and (5). Then $\hat{\kappa}$ was calculated by Eq. (9) or Eq. (10). These parameters are presented in Table 1. We dealt with the values of $\bar{\theta}$ carefully, because there are two solutions of $\bar{\theta}$ between 0° and 180° in Eq. (4). By comparing the two values of $\bar{\theta}$ with the mode and the antimode of the histograms, we judged which solution was the correct one. As a test of the assumptions of our model, we continue as follows.
5. The probability density function for the bipolar von Mises distribution was estimated for specimens B–D by substituting μ_0 and κ by $\bar{\theta}$ and $\hat{\kappa}$, respectively, into Eq. (1). The curve is depicted in Fig. 2. Then f_j^* was calculated by the integration of the function multiplied by n as

$$f_j^* = n\int_{10(j-1)^\circ}^{10j^\circ} \frac{1}{\pi I_0(\hat{\kappa})} \exp[\kappa \cos 2(\theta - \bar{\theta})]\,\mathrm{d}\theta. \tag{15}$$

6. For specimens B and C, the χ^2-test of the distribution was performed by substituting the above f_j^* into Eq. (14) with the null hypothesis that the measured data fit the von Mises distribution. The number of degrees of freedom is 17. The null hypothesis is not rejected with the critical region of 0.05.
7. For specimen D, the above f_j^* cannot be applied for the test, because some of f_j^* are less than 4: according to Batschelet (1981, p. 72) the χ^2-test needs $f_j^* \geq 4$. Combining the ten classes between −90° and 10° into one class, we performed the χ^2-test with a critical region of 0.05. The numbers of classes and number of freedoms are nine and eight, respectively. The null hypothesis was found not to be rejected.
8. We conclude that the measured data of specimens B–D fit the distribution well. Their calculated values of $\bar{\theta}$ and $\hat{\kappa}$ (Table 1) are significant.
9. The orientation of the mineral lineation for specimens B–D is represented by their mean direction ($\bar{\theta}$). The confidence interval (d_0) at the critical region of 0.05 (i.e. 95% confidence) is calculated by Eq. (12).
10. The estimated concentration parameter ($\hat{\kappa}$) is largest for specimen D and smallest for specimen B. A larger $\hat{\kappa}$ indicates a stronger degree of preferred dimensional orientation, in harmony with the impression gained by eye.
11. We prefer $\hat{\kappa}$ to $\bar{R}$ as an indicator of the degree of preferred dimensional orientation. The value of $1/\sqrt{\hat{\kappa}}$ is regarded as a parameter like the standard deviation of the Gaussian distribution for linear data (e.g. Mardia, 1972, p. 60). Thus, $\hat{\kappa}$ is closely related to the shape of the histogram of θ_i. A larger value of $\hat{\kappa}$ indicates a narrower and steeper peak of the histogram. Since the range of $\bar{R}$ is restricted between 0 and 1, $\bar{R}$ is not sensitive when $\bar{R} > 0.8$. On the contrary, as the range of $\hat{\kappa}$ extends from 0 to ∞, its sensitivity does not decrease where $\bar{R} > 0.8$.
12. Fig. 3 shows how $\bar{\theta}$ and $\bar{R}$ vary with increasing n. The curves become stable where $n > 50$. This gives a guide of n for their reliable estimation.

5. Summary

Statistical analysis of two-dimensional orientation data enables us to quantify the degree of preferred dimensional orientation using the concentration parameter of a bipolar von Mises type distribution. Three important points emerge from our analysis of mineral lineation in natural specimens.

1. This method provides us with an unequivocal method to judge whether the specimen has a mineral lineation or not. If the χ^2-test of uniformity for a set of orientation data does not reject the null hypothesis, we should conclude that the specimen has no significant lineation.
2. If the χ^2-test of the von Mises distribution does not reject the null hypothesis, we can determine the preferred orientation and its confidence interval by using Eqs. (4) and (12).
3. The degree of preferred orientation can be represented by the concentration parameter $\hat{\kappa}$. Although this parameter is only descriptive, it may be useful for the quantitative structural analysis of deformed rocks. How $\hat{\kappa}$ is related to strain and stress may be a target of future study.

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